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T4 ________	**E**	F4 ________

2016 Mathematical Contest in Modeling (ICM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

How to solve water scarcity

Summary

In this paper, the Logistic model of population forecast, gray theory, neural network model, regression analysis and curve fitting are used to analyze and discuss the region of North China where is in heavily lack of water resources.

Firstly, the water demand model and water supply model based on Logistic population prediction and GM(1,1) are established. Then for the selected region of North China, the current situation of water resources and causes of formation are stated both the social and environmental drivers by addressing physical and economic scarcity. For Task 3, total water demand in 15 years in the provinces of North China can be judged and calculated using per capita of water requirement. We can draw the conclusion that the total gap in North China is about 35.8 billion cubic metres by MATLAB, also the explanations and impacts on residents have been analysed in this section. Moreover, the result obtained by gray prediction before are tested with the application of Neutral Network Model and shows the result is relatively accurate.

Secondly, an intervention plan is designed with the intention of handling water scarcity, which mainly includes building reservoirs, South-to-North Water Diversion Project and protection of water resources. The number of reservoirs that should be built are determined by the demand-supply gap of water in the provinces. Considering Bohai is close to China, Hense water supply can be carried out from two aspects: water transfer project and desalination plants. Nonlinear programming theory is used to analyze the example of Beijing. A mathematical model can be built to determine the pollution degree of four provinces in North China. Regression analysis and curve fitting are used to deal with the forecasting of sewage disposal in the provinces.

Finally, the overall strengths and weaknesses of the plan as well as the effect on the surrounding areas and the entire water ecosystem are synthetically discussed.

Keywords: Water Scarcity, Gray Prediction Model, BP Neutral Network Model, Intervention Plan, Regression Analysis, Curve Fitting

Contents

1 Introduction

1.1 The introduction of water resources in China

Water is believed as the key element for life on earth. With the development of society, the conflict between water resources and human life has become a constant topic. The total amount of water resources on the earth is about 13.8 billion cubic kilometer, and 97.5% is sea water (about 13.45 billion cubic kilometer), freshwater only accounts for 2.5%, the vast majority of it is polar ice, snow glaciers and groundwater. The available water for the enjoyment of human beings is only 0.01%. In the 20th century, the population of the world has increased by two times, and water for human consumption five times, many countries in the world are facing water crisis.

As a country of great population, the situation of water resources are particularly grim. China's total amount of water resources is 2.8 billion cubic kilometer, ranking the sixth in the world, while the per capita share of it is 2240 m^3, about 1/4 of the world's per capita, ranking 109th in the world. China has been listed as one of the 13 water poor countries in the world. Currently, there are sixteen provinces' per capita water resources (not including transit water) under the serious lack of water-line, six provinces and autonomous regions (Ningxia, Hebei, Shandong, Henan, Shanxi, Jiangsu) per capita water resources amount less than 500 m^3. The shortage of water resources have become a deep concern of the government and the people in our country.

The reason why China is suffering water scarcity can be explained from several aspects:

Firstly, the regional distribution of water resources in China are uneven, the water and soil resources are not matched: the area of the Yangtzi River Valley and its Southern parts just accounts for only 25.5% of the national total, the total of water resources is 81%. The area of the Huaihe River Basin and its northern parts have a percentage of 63.5% in the national total, while the total of water resources is just 19%.

Next, the uneven distribution of water resources and the frequent occurrence of drought and water logging disasters have important impacts on water scarcity: The amount of precipitation for 4 months accounts for more than 70% of the whole year in most areas and it is more common with continuous high flow or the continuous dry years.

Thirdly, what is more urgent and serious to be solved is that the degree of China's water pollution has reached the warning line: with the process of urbanisation, the quantity of sewage discharged is rising sharply, while the ability to deal with urban sewage is increasing slowly.

1.2 Restatement of the Problem

Water on the earth is the most widely distributed and most important material. Humans require water resources for industrial, agricultural and residential purposes. However, according to the United Nations, 1.6 billion people (one quarter of the world's population) experience water scarcity. The present situation of water scarcity in the world is to be solved urgently. There are two primary causes for water scarcity: physical scarcity and economic scarcity. Meanwhile, human population increase or increasing rates of industrial consumption also places increased burden on the supply of fresh water.

The problems needed to be solved in this paper are listed as follows:

- Task 1: Develop a model that provides a measure of the ability of a region to provide clean water to meet the needs of its population. Noted that the dynamic nature of the factors that affect both supply and demand should be considered in our modelling process.

- Task 2: Pick one country or region where water is either heavily or moderately overloaded and explain why and how water is scarce in the region. We are required to explain both the social and environmental drivers by addressing physical and economic scarcity.

- Task 3: Use our model from Task 1 to show what the water situation will be in 15 years in the chosen region from Task 2, then analysis how the situation impact the lives of citizens of the chosen region.

- Task 4: Design an intervention plan taking all the drivers of water scarcity into account for the region we have chosen and discuss the impact on the surrounding area and the entire ecosystem. What's more, the overall strengths and weakness of the plan needed to be analysed as well as the function of our plan to mitigate water scarcity.

- Task 5: Use the intervention designed in Task 4 and the model built to the availability of project water into the future. Validate that whether the chosen region will become less susceptible to water scarcity and if will, when to occur with the corresponding scarcity.

2 Problem Analysis

The requirements of the subject are to provide access to clean, fresh water to help solve the world's water problems. Five main sub-tasks are listed as follows:

1. Task 1 is required to build a model to provide a measure of the ability of a region to provide clean water to meet the needs of its population. In our paper, the model of water demand and the model of water supply in population prediction are respectively built using the per capita of water requirement to calculate and judge.

2. For the task 2, according to the distribution of water shortage area in China, we can find that most provinces in North China (Beijing, Tianjin, Hebei, Shanxi) are water shortage areas, the reasons why and how water is scarce in this region are needed to explain.

3. In the task 3, we should use the model built in Task 2 to compute and validate the chosen regions in Task 1 and analyse the corresponding results. A related consideration is to incorporate the environmental drivers' effects on the model components.

4. An intervention plan aimed at water scarcity is designed in Task 4. Intervention measurements including desalination plants, building reservoirs, South-to-North Water Diversion Project and protection of water resources are put forward

in this article. What's more, we will discuss the overall strengths and weakness of the plan and the impact on the surrounding areas of North China as well as the entire water ecosystems.

5. We are required to use the designed intervention plan combined with our model to plan and coordinate water resources in the chosen provinces of North China and the purpose is to reduce the impact of water scarcity and meet the requirements of sustainability.

3 Assumption

(1) Assume that there will be less abnormal phenomenon such as earthquake, flood and other meteorological disasters in 15 years in the areas of North China.

(2) There will not be huge changes of per capita of water usage.

(3) The costs of reservoir construction in library capacity of provinces on average are basically the same.

(4) Without considering the effects of global warming and climate change on the quantity of fresh water.

(5) Ignore the management costs of Water Transfer Project in the late stage.

(6) Ignore the effects of other pollutants on waste water when calculating the extent of water pollution.

(7) There is no extra consideration in cost of water transportation when calculating the cost of desalination plants.

4 Symbols and Definitions

Tab 1: Variable Definition

Variable Symbols	Definition
P	The number of population (million people)
T	The year
α, β, κ	Constant
a	The grey number of development
b	The grey number of endogenous control
r	The Correlation Degree of series $\hat{x}^{(0)}(k)$ and $x^{(0)}(k)$
$\eta(k)$	The correlation coefficient of series $\hat{x}^{(0)}(k)$ and $x^{(0)}(k)$
ρ	The resolution
$\omega_{ij}(i=1,2;j=3,4,5)$	The weight of the input layer and hidden layer
$\omega_{jk}(i=3,4,5;k=6)$	The weight of the hidden layer and output layer
χ_1, χ_2	The input value of the Neural Network
y	The output value of the Neural Network
y_ρ	The teacherąŕs signal
e	The error between the actual output and the expectation in the Neural Network
λ	The Water saving level
Q	The total amount of water used in the region
GDP	The Gross product of the region

5 The Models

5.1 The forecast of water demand using population prediction model

In order to better establish the dynamic model of the supply of water resources and demand changes of water resources with population growth, the model that can predict how much water is needed in a certain region in 15 years should be built firstly. With current water supply capacity of the region, the quantity of water supply in provinces can be forecasted, the difference between the amount of water supply and water demands is the gap to be filled by a strategic plan. With respect to per capita of water usage annually, it is regarded as constant in the case of no large-scaled water-saving. Thus the amount of water usage is decided by the population of relative provinces. Meanwhile, because of the real situation which the speed of population growth has slowed and will not keep increasing, the Logistic model of population forecast is applied to compute the population in 2030. For water requirement, it can be predicted by the prediction model under the condition of the available water resources. The model of Logistic function is as follows:

$$P = \frac{k}{1 + \alpha e^{-\beta t}} \tag{5.1}$$

Where P is the number of population, t represents the year and α, β, k are obtained constant.

5.2 The prediction model of water supply based on GM (1,1)

In this paper, the grey forecasting model [3] is used to predict the water supply capacity of the provinces of North China (Beijing, Tianjin, Hebei, Shanxi), and the specific algorithm process is as follows:

Step 1: Assume that the original series $x^{(0)}$ = ($x^{(0)}$(1), $x^{(0)}$(2), ... , $x^{(0)}$(n)), to weaken the randomness and volatility of the original series and provide more efficient information for the Grey Forecasting Model, we would do some preprocessing to the original data before building the Grey Forecasting Model,usually we use the one-time accumulated generating to series $X^{(0)}$, which is 1-AGO (Accumulating Generation Operator), then note that the generation series is:

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)) = (x^{(0)}(1), x^{(1)}(1) + x^{(0)}(2), ..., x^{(1)}(n-1) + x^{(0)}(n)) \tag{5.2}$$

Step 2: GM(1,1) is a dynamic model built by the first order differential equation with single variables:

$$x^{(0)}(k) + az^{(1)}(k) = b(k = 1,2,3,...,n) \tag{5.3}$$

Here we do mean generation with consecutive neighbors on $x^{(1)}$ to generate $z^{(1)}$, that is

$$z^{(1)}(k) = 0.5[x^{(1)}(k) - x^{(1)}(k-1)] \tag{5.4}$$

$$z^{(1)}(k) = (z^{(1)}(2), z^{(1)}(3), ..., z^{(1)}(n)); k = 2,3,...,n \tag{5.5}$$

Step 3: The winterization equation(also known as shadow equation)of the dynamic model in Step 2 is:

$$\frac{dP}{dt} = rP(1 - \frac{P}{K}) \tag{5.6}$$

The efficient section of a is(-2,2): Use the Least Square Method to solve the parameter series

$$\hat{a} = [a,b]^T = (B^T B)^{-1} B^T Y \tag{5.7}$$

as:

$$\hat{a} = (a,b)^T = (B_T B)^{-1} \cdot B^T \cdot Y_n \tag{5.8}$$

Where

$$V(t) = \frac{S}{C\exp(-\mu t) + 1} \tag{5.9}$$

$$Y_n = [x_{(0)}(2), x_{(0)}(3), ..., x_{(0)}(n)]T \tag{5.10}$$

Step 4: Determine the time response series of model GM(1,1) as:

$$\hat{x}(k+1) = [x_{(0)} - \frac{b}{a}]e^{-ak} + \frac{b}{a} \tag{5.11}$$

Step 5: Obtain the value of simulation of$x^{(1)}$

$$\hat{x}^{(1)} = (\hat{x}^1(1), \hat{x}^1(2), ..., \hat{x}^1(n)) = (x^{(0)}(1), x^{(1)}(1) + x^{(0)}(2), ..., x^{(1)}(n-1) + x^{(0)}(n)) \tag{5.12}$$

Step 6: Reduce the value of simulation

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{5.13}$$

Step 7: It's necessary to conduct Residual Test, Relational Coefficient Test and Posterior-Variance-Test to ensure the high prediction precision and believable degree.

(1)Residual Test

Obtain the residual series $e^{(0)}(k)$ of $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ respectively, the relative error series $\triangle_k$ and average relative error $\overline{\triangle}$:

$$e^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k) \tag{5.14}$$

$$\triangle_k = |\frac{e(k)}{x^{(0)(k)}}| \times 100\% \tag{5.15}$$

$$\overline{\triangle} = \sum_{k=1}^{n} \triangle_k \tag{5.16}$$

(2)Posterior-Variance-Test

Obtain the average$\overline{x}$of original data and residual average $\overline{e}$:

$$\overline{x} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k) \tag{5.17}$$

$$\overline{e} = \frac{1}{n-1} \sum_{k=2}^{n} e^{(0)}(k) \tag{5.18}$$

Obtain the variance of original data s_1^2, the residual variance s_2^2, the mean square ratio C and small error possibility P :

$$s_1^2 = \frac{1}{n}\sum_{k=1}^{n}[x^{(0)}(k) - \overline{x}]^2 \tag{5.19}$$

$$s_2^2 = \frac{1}{n-1}\sum_{k=1}^{n}[e^{(0)}(k) - \bar{e}]^2 \tag{5.20}$$

$$C = s_2/s_1, P = p\left\{\left|e^{(0)}(k) - \bar{e}\right| < 0.6745 s_1\right\} \tag{5.21}$$

Let $\xi_k = \left|e^{(0)}(k) - \bar{e}\right|$, $s_0 = 0.6745 s_1$, then $P = p\{\xi_k < s_0\}$;

According to the Gray System Method,generally the smaller $e^{(0)}(k)$, Δ_k and Care,the larger the number of P will be, thus the model precision will be higher. The first step prediction precision of GM(1,1) will be above 98% , the second and fifth step will be above 97% when the grey number of development $a\epsilon[-0.3, 2)$,which can be used in long term prediction.

Step 8: Calculate the simulation value if the precision meets the requirement.

5.3 The reason to account for choosing North China to analyse

Fig.1 shows the regional distribution with lack of water resources in World and China respectively. From the figure, we can conclude that China is facing the shortage of water resources, especially heavily in North China and Eastern Coastal Areas of China, which are close to our life. To better understand the grim situation faced by water resources, advocate people around to save water, improve the awareness of water conservation, North China is selected to analyse in our paper.

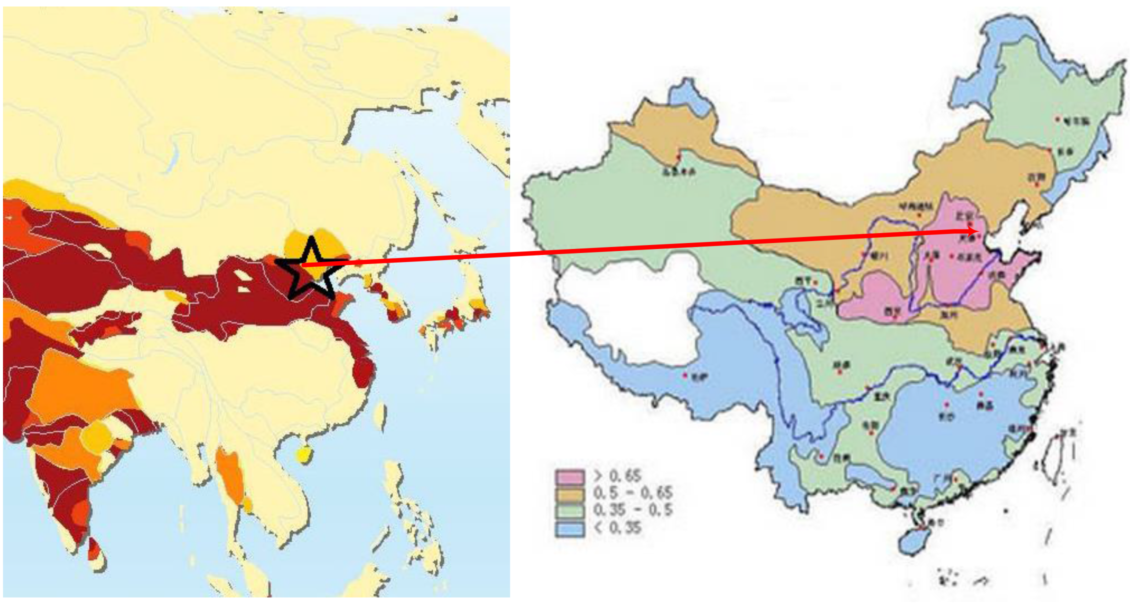

Fig 1: The regional distribution with lack of water resources in World and China

The first step is to find out the data of total water resources and water-using quantity from 2005 to 2014 in various regions. Considering a relative stability in total water resources, the average of total water resources in corresponding areas are used to be predicted values in 2030.

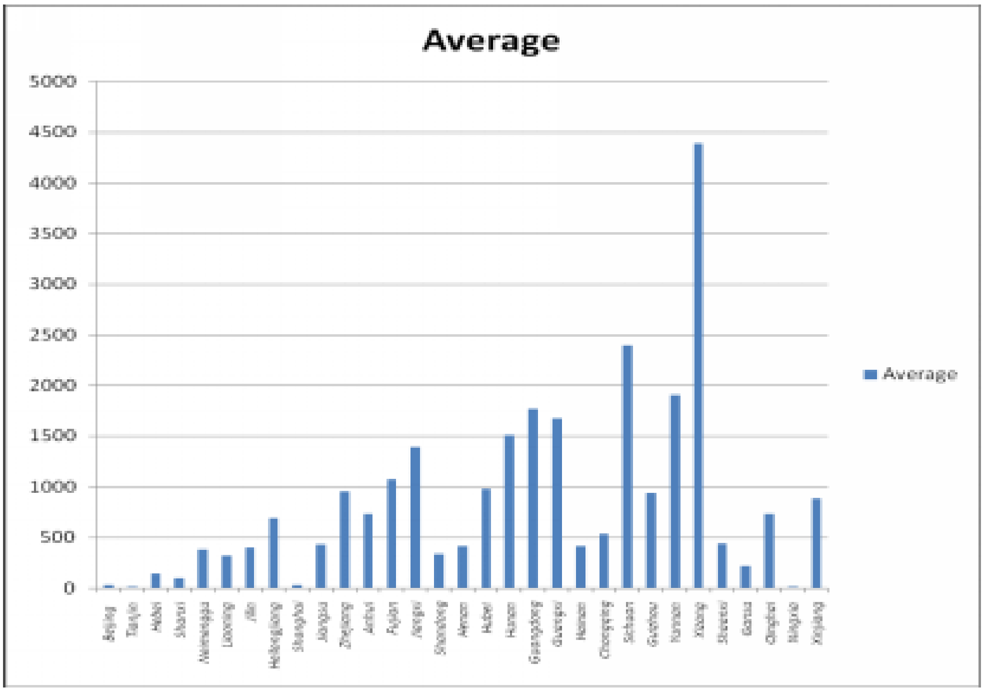

Fig 2: The average value of total water resources in different regions

From the analysis of Fig.2, the total water resources in Xizang, Sichuan, Yunnan and *et.al* are relatively numerous, but Shanghai, Beijing, Tianjin and *et.al* are relatively inadequate. By searching for relevant documents, the reason to explain the present situation of water scarcity in North China can be discussed from physical aspects and economic aspects.

5.3.1 Physical scarcity

- The water resources of our country are mail-distributed in space and time, which is most responsible for lacking of water in North China. Geographically, the general trend is fewer and fewer from south to north and from east to west. The population of four provinces in North China (Beijing, Tianjin, Hebei, Shanxi) accounts for 24.0% of the national total, the areas of farmlands account for 45% and the industrial production in the Beijing-Tianjin-Tangshan area has a percentage of 10% in the total industrial output value. However, it is noteworthy that the amount of water resources accounts for only 6% of the country. From the point of seasonal distribution of precipitation, the rain in North China is highly concentrated in July - August and the amount of precipitation in these two months has a share of about 80% in the annual total amount. The continuous arid occur in Spring and Winter. What's more, there is a great change in the yearly precipitation in North China.

- Ecological environment and the increasing frequency in droughts have a negative effect on water scarcity. According to statistics, on the one hand, the forest coverage rate is only 3.5% in North China, which not only leads to the poor ability of water conservation, seriously water loss and soil erosion and the reduction of groundwater, but also result in the decrease of air humidity and increase frequency of droughts. On the other hand, to meet the productive and domestic water supply, a large amount of groundwater exploitation is adopted in the North China Plan, which makes the groundwater lowering a great deal in large scope and contributes to the formation of the largest of funnel-shaped zone of groundwater currently.

5.3.2 Economic scarcity

- The rapid growth of population and development of industrial and agriculture have exacerbated the burden on the tension of water resources. First, since ancient times, North China has been one of the most concentrated areas of population distribution. The density of population in four provinces rank in national provinces in front. Second, from the initial foundation period of new China, the planting areas of farm crops continue to increase, irrigated area constantly expand and agricultural water consumption increase exponentially. Beijing-Tianjin-Tangshan area has become the largest comprehensive industrial base in the north of China and the supply of fresh water is even more tense.

- Water comprehensive utilization ratios are low and the situation of waste and environmental pollution is serious. Water conservancy projects in North China, especially the agricultural irrigation projects do not match well. Meanwhile, facilities of leakage and seepage are not perfected, different degrees of seepage and leakage become existing phenomenon, they are perhaps no surprise that the effective utilization of water resources is only about 50% and the loss rate of water supply in cities is up to more than 10%. Moreover, a large number of industrial waste water is discharged into rivers, which causes pollution of water quality at different degrees, particularly the pollution of sea water and river water is the most serious, in simple terms, its downstream of the river can not be directly used as production and living.

5.4 Forecast of water demand and water supply in North China in 2030

5.4.1 Prediction of water demand

For the four provinces of North China, the number of population of four provinces in 2005 to 2014 can be derived from data attached list one, doing regression analysis with Logistic MATLAB model and getting the results, as shown in Fig.2:

From Fig.3, we can see, the population growth speed in Hebei and Shanxi have no decreasing trend in the short term, closely related to the reality of its development stage in the two provinces. The saturation in Beijing and Tianjin will bring greater pressure of population to Hebei and Shanxi, will also bring new challenges on water resources. The population in Beijing and Tianjin will grow slowing 2020, gradually stabilized, there is no increase in population. But in 2030, the population is still increasing, making water resources more tense.

The formulas obtained in each province are as follows:

$$Beijing : P = \frac{3601}{1 + 1.64\mathrm{e}^{-0.062\mathrm{t}}} \tag{5.22}$$

$$Tianjin : P = \frac{2532}{1 + 1.73\mathrm{e}^{-0.061\mathrm{t}}} \tag{5.23}$$

$$Hebei : P = \frac{14068}{1 + 1.1\mathrm{e}^{-0.0128\mathrm{t}}} \tag{5.24}$$

$$Shanxi : P = \frac{7148}{1 + 1.2\mathrm{e}^{-0.016\mathrm{t}}} \tag{5.25}$$

Seeking out the number of provinces in 2030, and getting the water required of every province in 2030 as shown in Tab.2 by the per capita consumption of provinces and cities [4].

Tab 2: Water requirement of all provinces in North China in 2030

Variable Symbols	Definition			
Province	Beijing	Tianjin	Hebei	Shanxi
The number of population (million people)	2433	1839	7821	3961
Per capita water consumption(m^3)	172	161	260	203
Water requirement(billion cubic kilometer)	41.8	29.6	203.3	80.4

5.4.2 Forecasting and gap of water supply

In order to get the gap of water resources in 2030, supply quantities of various provinces and cities in North China are predicted in this paper by the establishment of grey prediction model GM(1,1) in Section 5.2 and the data in Schedule 2. The results as shown in Fig.4:

The final predicted value of water supply in various provinces in 2030 is shown in Tab.3. At the same time, it can be seen from Fig.4 that the water supply in Hebei province has a clear downward trend, which is caused by the development of industrial water pollution and the imperfect water resources management system. Shanxi has a slight rise, which is prone to drought, the rose of this year's water

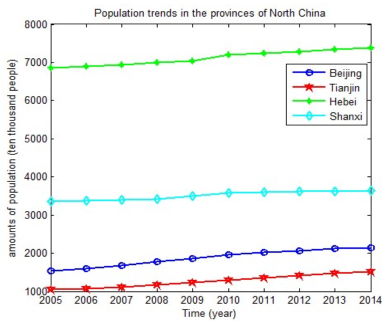

Fig 3: The trend chart of population in the provinces of North China

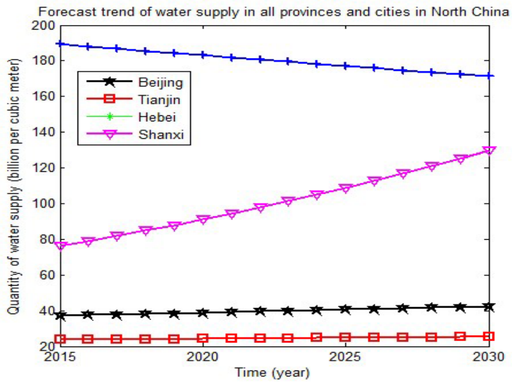

Fig 4: The forecast trend of water supply in the provinces of North China

Tab 3: The quantity of water supply in the provinces of North China in 2030

Province	Beijing	Tianjin	Hebei	Shanxi
Water supply in 2030(billion cubic kilometer)	37.6	27.1	176.1	78.5

supply is contributed by the implementation of the Huang Jijin project. There is no change in Tianjin and Beijing. Basically, the amount of water supply in North China area is of a certain stable range, will be sustainable supplied in the near future. But it is known by the situation of population and demand, the water resources will be unable to meet the needs of production and daily life with the increase of population and rising demand for water. If water supply keeps the same as the original level and the law of development, without any measures, the situation will be more and more severe. The resulting gap is shown in Fig.5:

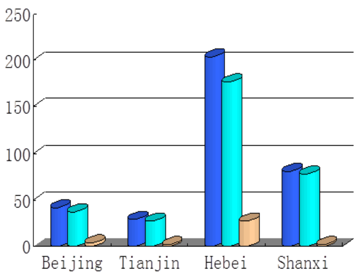

Fig 5: The supply and demand of water resources in North China

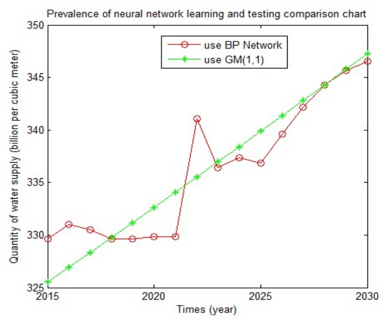

Fig 6: The result comparison diagram

As is shown in Fig.5, the total amount of water gap in North China is 35.8 billion cubic meters, if there is no effective strategy of water resources program taken in, Hebei will face great shortage of water resources by 2030. Beijing, Tianjin and Shanxi will also face varying degrees of water shortage soon. The shortage of water resources will restrict the development of industrial and agricultural production in North China, and affect the normal production and life of people.

5.4.3 The advantages and disadvantages of the model

Advantages: The Gray Model would have some errors if the data is not that enough. We can adjust the parameters to minimize the error. The Gray Model's timeliness is limited and not suitable for long term prediction or analysis as its characteristics are having little parameters and small fault tolerance, rapid attenuation and increasing. We take it as our model considering that we have little usable data and time points, which would result in lots of unsure factors.

Disadvantages: The disadvantage on the model is that it can not show the lower growth rate. The error of the model is relatively big comparing the prediction result with the actual result, so there should be large space of improvement such as changing the Gray Model into the Double Exponential model to minimize the error. And it would also be helpful to use more data, then make prediction by the fitting data.

5.4.4 Testing on the model

BP(Back Propagation) Neural Network[5], the learning process on the error back-propagation algorithm(BP), including the forward propagation of information and the back propagation of the error. The neurons in input layer are in charge of accepting the input information from outside and transfer them to the neurons in the middle layer; the middle layer is inside information treatment layer which is in charge of information transform and can be designed as Single Hidden Layer or Multiple Hidden Layer structure based on the requirement on the ability of information transform. The information from the last hidden layer to neurons on other layers, complete a learning forward propagation treatment process after being further processed, then output the result through the output layer. The process will come into the back propagation of error when there is difference between the expectation and actual output. The error would be modified the weight in all the layers based on the error gradient-decreasing through output layer and back propagate to hidden layers and input layers. The forward propagation of information and the back propagation of the error from balance to imbalance are the process to adjust weight in all layers and the Neural Network to learn. The process won't stop until we get the accepted error or the presupposition learning time. A simple Neural Network is shown as follows:

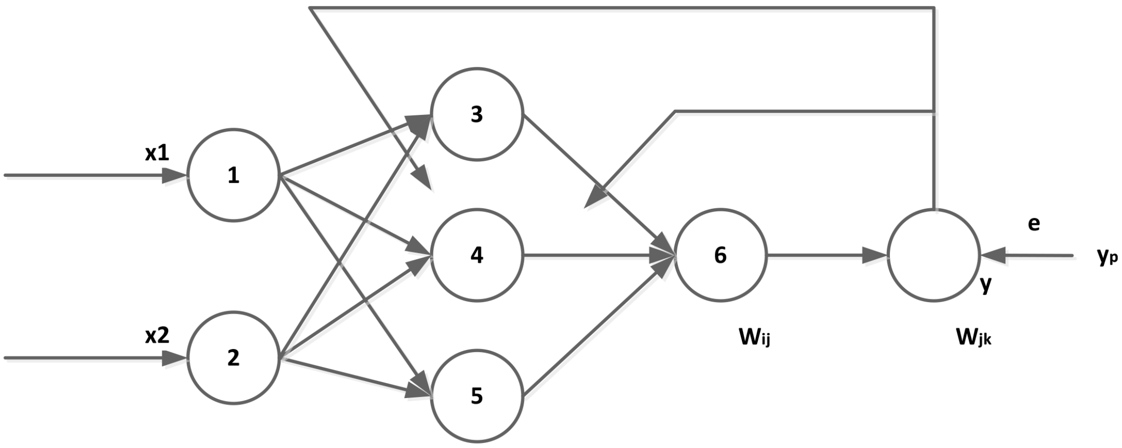

Fig 7: The structure diagram of the Neural Network

The node 1 and 2 are input layers, node 3, 4 and 5 are hidden layers, node 6 is output layers. The weights between input layer and output layer are $w_{13}, w_{14}, w_{15}, w_{23}, w_{24}, w_{25}$ in order, the weights between the hidden layers and output layers are w_{36}, w_{46}, w_{56}, the subscript is the number of node. The threshold between the hidden layers and output layers are $\theta_3, \theta_4, \theta_5, \theta_6$

(1)Feedforward calculating

Assume that the input and output of node j are

$$I_j = \sum_{i=1}^{N} w_{ij} \cdot O_i and O_j = f(I_j) \tag{5.26}$$

Where $f(I_j)$ is the incentive function

$$f(I_j) = \frac{1}{1 + e^{-I_j}} \tag{5.27}$$

Because the output of the hidden layers is the input of the output layers, the total input and output of the kth node in output layer are:

$$I_k = \sum_{j=1}^{H} w_{jk} \cdot O_j and y_k = O_k = f(I_k) \tag{5.28}$$

(2)Weight adjusting

Define the error function as:

$$E_p = \frac{1}{2} \sum_{k=1}^{M} (d_k - y_k)^2 \tag{5.29}$$

To simplify the problem, the following calculating all focus on the nodes respectively. Note the error function E_p as E.

(a)Weight adjusting in output layers

The modified weight formulas:

$$\Delta w_{jk} = -\eta \frac{\partial E}{\partial w_{jk}} = -\eta \frac{\partial E}{\partial I_k} \frac{\partial I_k}{\partial w_{jk}} \tag{5.30}$$

Define the back-propagation error signal as:

$$\delta_k = -\frac{\partial E}{\partial I_k} = -\frac{\partial E}{\partial O_k}\frac{\partial O_k}{\partial I_k} \tag{5.31}$$

$$\frac{\partial O_k}{\partial I_k} = \frac{\partial f(I_k)}{\partial I_k} = f'(I_k) \tag{5.32}$$

$$f'(I_k) = f(I_k)[1 - f(I_k)] = O_k(1 - O_k) \tag{5.33}$$

So

$$\delta_k = (d_k - O_k)O_k(1 - O_k) and \frac{\partial I_k}{\partial w_{jk}} = \frac{\partial}{\partial w_{jk}}(\sum_{j=1}^{H} w_{jk}O_j) = O_j \tag{5.34}$$

Thus we can obtain the modified formula of any Neural Network's weight in output layers:

$$\Delta w_{jk} = \eta O_k(1 - O_k)(d_k - O_k)O_j \tag{5.35}$$

(b)The correction on the weight in hidden layers:

$$\Delta w_{ij} = -\eta\frac{\partial E}{\partial w_{ij}} = -\eta\frac{\partial E}{\partial I_j}\frac{\partial I_j}{\partial w_{ij}} = -\eta\frac{\partial E}{\partial I_j}O_i \tag{5.36}$$

where

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}}(\sum_{i=1}^{N} w_{ij} \cdot O_i) = O_i \tag{5.37}$$

It can't be obtained directly as there is no direct function relationship between the error function E and hidden layer input I_j , thus

$$-\frac{\partial E}{\partial I_j} = -\frac{\partial E}{\partial O_j}\frac{\partial O_j}{\partial I_j} = \sum_{k=1}^{M}(-\frac{\partial E}{\partial I_k})\frac{\partial}{\partial O_j}(\sum_{j=1}^{H} w_{jk} \cdot O_j) \cdot f'(I_j) = (\sum_{k=1}^{M} \delta_k w_{jk}) \cdot f'(I_j) \tag{5.38}$$

The back propagation error signal in hidden layer is

$$\delta_i = f'(I_j) \cdot \sum_{k=1}^{M} \delta_k w_{jk} \tag{5.39}$$

From which we can obtain the modified formula of the weight in hidden layer

$$\Delta w_{ij} = \eta O_j(1 - O_j) \cdot (\sum_{k=1}^{M} \delta_k w_{jk}) \cdot O_i \tag{5.40}$$

To outburst the difference and efficient furthermore, we transform the official data to the actual number of people who have been infected every week. And test the prediction error and precision of GM (1,1) using BP Neural Network and the Gray prediction. The results are as follows:

It is obvious that the results of the GM (1,1) and the Neural Network are close to each other, which prove the correctness of Model 1 furthermore.

Tab 4: Results comparison in 16 years

Raw data		Data to predict		
Year	Data	Year	BP Neural Network	GM(1,1) prediction
2005	308.5	2015	325.525	329.593
2006	320.6	2016	326.932	330.997
2007	319.4	2017	328.345	330.495
2008	309.3	2018	329.765	329.609
2009	308.9	2019	331.19	329.601
2010	315.2	2020	332.622	329.812
2011	315.2	2021	334.06	329.812
2012	327.7	2022	335.504	341.066
2013	325.3	2023	336.954	336.441
2014	325.8	2024	338.411	337.386
–	–	2025	339.874	336.861
–	–	2026	341.343	339.606
–	–	2027	342.819	342.201
–	–	2028	344.301	344.258
–	–	2029	345.789	345.671
–	–	2030	347.284	346.548

5.5 The problem analysis with an intervention plan

5.5.1 Intervention plan 1: Building reservoirs

Water storage includes two aspects: groundwater and surface water. Changing environment constantly and increasing vegetation coverage are needed for the storage of groundwater, it is difficult to achieve in the short term. Therefore, we build reservoirs to increase the storage of surface water primarily, in order to make up for the gap of water supply and demand gap in 2030. First of all, we determine the number of reservoirs should be built through the supply and demand of various provinces' water supply and demand. Then evaluate of the effectiveness of the construction of the reservoirs through the impact of their flood control, power generation, aquaculture and other aspects. Tab.5 can be gathered by looking for the reservoirs' capacity and quantity in provinces and cities.

Tab 5: Number and capacity of reservoirs in four regions

Regions	Beijing	Tianjin	Hebei	Shanxi
Large reservoir capacity (billion cubic meters)	36.68	7.76	56.48	6.13
The quantity of large reservoir	4	3	18	6
Average library capacity (billion cubic meters)	9.17	2.59	3.14	1.02

From Tab.5, it can be seen that the area reservoir capacity in each area is not identical, it is related to different geographical factors and development strategies. Therefore, we should base on average library capacity of each area when we are calculating the number of reservoirs should be built in the region. Shanxi, the amount of water shortage is 1.9 billion cubic meters in 2030, while the average reservoir capacity of the large reservoir is only 1.02 billion cubic meters. According to the definition, the reservoir which storage capacity is more than 1 million cubic meter is called large reservoir, 1 million to 1 billion called medium-sized reservoir. As a result, a large reservoir with a capacity of 1.1 billion cubic meters and a medium-sized reservoir with a capacity of 60 million cubic meters can be built in Shanxi.

According to the average reservoir capacity, the number and capacity of the reservoir should be built in each province, that is, The quantity and capacity of the reservoirs will be built in various provinces is as Tab.6, based on the average reservoir capacity.

Tab 6: Number and capacity of reservoirs to be built in four areas

Regions	Beijing	Tianjin	Hebei	Shanxi
Reservoir water storage (billion m^3)	4.2	2.5	27.2	1.9
Average reservoir capacity (billion m^3)	9.17	2.59	3.14	1.02
The quantity of the construction reservoir (C)	1	1	9	2

To solve the problem of water shortage by building a reservoir to increase water storage capacity, it can also bring huge benefits. Building a reservoir to increase water storage capacity can not only solve the problem of water shortage, but also can bring huge benefits. For example, the construction of Zhangfeng Reservoir in 2005, of which generating capacity is up to 770, meanwhile, flood control standard from once every 5-10 years is increased to once in 20 years, enhancing the flood control capacity greatly. Therefore, the pressure on the entire North China can be decreased by building reservoirs in the four regions. Meanwhile, it can also save energy and protect the environment by turning thermal power generation into hydropower. In addition, the ability of flood control can be enhanced by increasing the quantity of reservoirs. Precipitation in North China is seasonal changing, the measures to better storage of precipitation and avoiding precipitation of wasted and seasonal shortage of water. Last but not least, we can also improve economic efficiency through aquaculture, etc. There will be some risk when building reservoirs inevitably. The general service life of reservoir is about 50 years. As a result, the water can only be fully guaranteed within 50 years by building reservoirs. With time goes on, repairing measures should be taken to solve the re emergence of water supply and security issues.

5.5.2 Intervention plan 2: South-to-North Water Diversion Project

Through the study on the storage of water resources we can find that, though certain ecological benefits and economic benefits similar to electric power are contributed by the reservoirs, a big problem is that the sustainability is poor for a reservoir is available just for 50 years. Thus the flow problem of water resources will be discussed in this section, making up the shortcomings of reservoirs based on existing desalination plants and Water Transfer Projects. Generally there are many flow manners of water resources, of those, the transfer project has a good advantage to provide water resources effectively and efficiently, like desalination plants and Water Transfer Projects. The transfer project in North China is centered on South-to-North Water Diversion Project, at the same time, Yellow River to Shanxi Province, Yellow River to Tianjin and Huang-Wei water transfer projects also have played a role. South-to-North Water Diversion Project is divided three sub-projects: eastern route, central route and western route. The most part of water transfer from eastern route and central route is supplied for North China and the upstream of Huanghe River will be provided water resources by western route. Also, North China is close to Bohai, desalination plants in Bohai is also an important part of the flow manner of water resources.

Take Beijing as an example. Beijing obtains water resources from eastern route and central route. The starting point of eastern route is Jiangsu and the starting point of central route is Hubei. Assume the new increased water supply by eastern route is represented by $\triangle w_1$, the new increased water supply by central route is represented as $\triangle w_2$. The nonlinear programming is shown as follows:

$$\min\left[\Delta\eta_1(\mathrm{bi},bj,d,1+\frac{\Delta \mathrm{w}_1}{\mathrm{w}_1})+\Delta\eta_2(bi,bj,d,1+\frac{\Delta \mathrm{w}_2}{\mathrm{w}_2})\right] \tag{5.41}$$

$$S.t.\Delta w_1+\Delta w_2=4.2 \tag{5.42}$$

$$\Delta w_1\geq 0,\Delta w_2\geq 0 \tag{5.43}$$

Among water transfer projects with the relatively large scale in Shanxi province currently, only Yellow River to Shanxi Province is in use, so all gaps can only be filled by the Inner Mongolia River in the Yellow River. Tianjin is that the same with Beijing for the two provinces' gaps are filled by eastern route and central route. Hebei has the largest gap and meanwhile, the number of available water transfer projects is larger than others with eastern route, central route and Huang-Wei water transfer projects.

5.5.3 Intervention plan 3: Desalination plants

Desalination plant is an important field to explore actively. Getting fresh water from the ocean, this worldwide issue has become a burgeoning industry. At present, the scale of desalination plants in China has reached 800 thousand tons per day. The technology and application level desalination plants in Tianjin belong to the first rank in China and now, the projects of water desalination have been built up to a total capacity of 21.7 ten thousand tons per day, accounting for 41.4% of the country. For the Seawater Reverse Osmosis Desalination Project of Da Guang Xin Quan with 10 ten thousand tons / day in Tianjin, prophase total investment is about one billion and 100 thousand tons / day of desalination water can be produced one times, moreover, after the expansion, the scale can be up to 150 thousand tons / day. According to the current situation of China's technology development, the desalination cost is between 4 and 7 yuan per square metre. Most importantly, with the possible application of new technologies such as steam distillation, dew point evaporation, the desalination cost will continue to decline in the future.

In order to meet the deficiency in water use in the next 15 years until 2030, assuming building new desalination plants, the investment costs including prophase total investment and desalination cost in 10 yeas can be calculated using Eq.(5.44):

$$\mu=(\frac{\Delta w}{I})S+\rho\Delta w \tag{5.44}$$

Where S represents prophase total investment, ρ represents desalination cost. In the example of Da Guang Xin Quan in Tianjin, make S 10 billion and the predicted value of ρ in 2030 is 2.82 yuan. Δw is deficiency in water use and I is the annual amount of desalination, about 0.55 billion cubic meters / year.

Of four provinces in North China, only Hebei and Tianjin are coastal cities, thus desalination plants should be constructed in these two provinces. Certain transition costs will be spent if desalinated water is provided to Beijing and Shanxi, but noted that four provinces can contact one another by rivers and the distance is small, the transition costs will be ignored when computing the investment cost model. Finally the result of total investment costs in covering the gap using desalinated plants is 752 billion, which stands well above the existing cost of water transfer project. Due to the ecological effects and characteristic unable to provide water permanently and sustainably in water transfer projection, in fact, the advantages of water diversion projects are more significant just to make up the gap in 15 years. However, if more sustainable and efficient development is considered, it is necessary to increase investment in desalination.

5.5.4 Intervention plan 4: Water resources protection and sewage treatment

It can be found that through the study of above of water flow that the cost of building reservoir is high while sustainability is poor. There are many using the existing water diversion project water resources flow cost is low, poor sustainability and environmental benefit is low, high cost of sea water desalination, sustainability, environmental benefits are higher. In recent years, the sewage treatment has become the object of consideration for its low cost and the characteristics of the environment protection. The sewage treatment will be discussed here. Environmental pollution, including the problems which lead to the production of sewage, water shortages, and other serious impact on the quality of water resources. For example, soil salinization, heavy metal pollution. It makes the sewage discharge and treatment very important, so we need to ease the problem by the means of building a sewage treatment plant. Meanwhile, enormous environmental benefits and social benefits can be gained through the use of reuse water, which can be fully used of. Illustrating the environmental benefits of pollution control firstly, considering the size of the degree of environmental pollution. In order to simplify the model, the content of ammonia nitrogen is used to measure the amount of pollutants, and establish the following formula [10]:

$$M = p \times c \tag{5.45}$$

$$Z = 1000e^{0.1M}/y \tag{5.46}$$

Where M means the total content of ammonia nitrogen in Wastewater, c means the tons of sewage treatment, p means the ammonia nitrogen content of sewage in every ton, Z means the population in the area.

The value of the objective function is greater, the more serious pollution in the area, the need to establish a sewage treatment plant is greater. The following is the prediction of sewage discharge on the provinces. The ammonia nitrogen in per ton of sewage is related to the type of sewage, but there is of little change in one province or city, so it can be considered to a constant in this paper. At the same time, we can calculate the values of each of the provinces and observe the pollution degree by using the prediction of population above, forecast of the sewage emissions and sewage ammonia nitrogen, in order to provide the basis on the necessity to build a sewage treatment plant.

The constant value of ammonia nitrogen content in the units of various provinces and cities as shown in Tab.7:

Tab 7: Amounts of ammonia nitrogen in sewage

Provinces	Beijing	Tianjin	Hebei	Shanxi
Ammonia nitrogen content of sewage in every ton	1.3	3.3	4.1	4.2

The regression analysis [11] is made to predict the total waste water in 2030 in the total discharge of sewage of the provinces in North China. Take Hebei Province as an example, draw the scatterplot and curve fitting. The results are as shown in Fig.8:

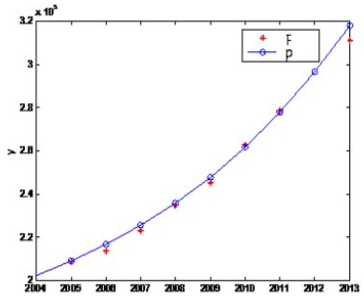

Fig 8: Fitting curve of total sewage in Hebei

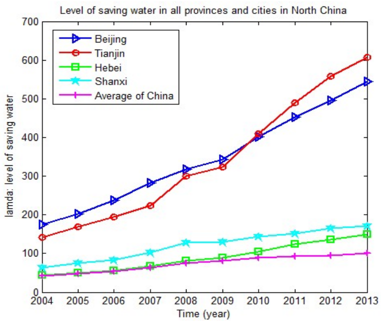

Fig 9: The water-saving level in North China and whole country

From Fig.8 it can be seen that the total discharge of sewage is year by year index rising trend in Hebei Province from 2004 to 2013. That means with the development of social economy and population, the industrialization deepening and living waste water increases. As a result, the sewage emissions continue to increase, great influence is caused on the environment, and the waste water treatment needs are increasing.

For the other three provinces, the four curves above obtained from the regression equation, in the same way. Following is the regression equations of Beijing, Tianjin, Hebei and Shanxi:

$$g_1(t) = 9.2 \times 10^4 e^{0.051(t-2003)} \tag{5.47}$$

$$g_2(t) = 4.8 \times 10^4 e^{0.050(t-2003)} \tag{5.48}$$

$$g_3(t) = 1.9 \times 10^5 e^{0.049(t-2003)} \tag{5.49}$$

$$g_4(t) = 8.5 \times 10^4 e^{0.046(t-2003)} \tag{5.50}$$

It can be predicted the total discharge of sewage that in 2030, as shown in Tab.8:

Tab 8: The discharge capacity of sewage in 2030

Provinces	Beijing	Tianjin	Hebei
The discharge capacity of sewage (ten thousand tons)	280750.52	147059.77	559845.9

6 Analysis and summary

Water is the key element for life on earth. With the development of society, the conflict between water resource and human life has become a constant topic. The total water resource on earth is about 13.8 billion cubic kilometer, and 97.5% is sea water (about 13.45 billion cubic kilometer), freshwater only accounts for 2.5%, the vast majority of it is polar ice, snow glaciers and groundwater, suitable for the enjoyment of human beings is only 0.01%. In the 20th century, the population of the world has increased two times, and human water increased 5 times, many countries in the world are facing water crisis.

As a country with large population, the water resources situation is particularly grim. China's total water resources are 2.8 billion m^3, ranking sixth in the world, while the per capita share of it is 2240 m^3, about 1/4 of the world's per capita, ranking 109th in the world. China has been listed as one of the 13 water poor countries in the world. Currently, there are sixteen provinces' per capita water resources (not including transit water) under the serious lack of water-line, six provinces and autonomous regions (Ningxia, Hebei, Shandong, Henan, Shanxi, Jiangsu) per capita water resources amount less than 500 m^3. The shortage of water resources has become a deep concern of the government and the people in our country.

$$\lambda = \frac{GDP}{Q} \tag{6.1}$$

Where λ is the level of water saving , Q is the total amount of water used in a region, GDP is the gross product of the region. According to data from the third schedule, the trend of the λ change in the four provinces of North China and in whole country is as Fig.9.

From Fig.9, in the provinces of North China, Beijing and Tianjin have the highest water-saving level, while the water-saving level in Hebei and Shanxi are generally higher than the national level, lower than the average level in North China. In addition, the development speed of the two provinces is slow, obviously investment in water-saving project of Hebei and Shanxi are relatively small. Overall, it is necessary to increase the investment in water conservation in Hebei and Shanxi to mitigate water scarcity of North China.

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